# THE BRAUER GROUP OF A CURVE OVER A STRICTLY LOCAL DISCRETE VALUATION RING

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ABSTRACT. Let $K$ be the field of fractions of a curve over $R$ where $R$ is the henselization of a regular local ring on an algebraic curve over a field which is algebraically closed and has characteristic 0. Then $K$ has the exponent = degree property for division algebras. In fact every central finite dimensional $K$-division algebra with exponent $n$ is a cyclic algebra of degree $n$.

In this paper we continue to investigate the structure of division algebras $D$ finite dimensional over their center $K$. The motivating problem is to classify those fields $K$ that have the *exponent = degree* property for division algebras. We say that $K$ has the *exponent = degree* property if for any central $K$-division algebra $D$ the exponent of the class $[D]$ in the Brauer group $\mathrm{B}(K)$ is equal to the degree $\sqrt{(D:K)}$ of the division algebra. Throughout this paper $k$ is an algebraically closed field of characteristic 0.

**Example 1.** Some fields that are known to have the exponent = degree property are listed below.

(1) A global field (an algebraic number field or a function field finitely generated of transcendence degree 1 over a finite field). This is classical.
(2) The quotient field of either (a) the henselization $\mathcal{O}^h_{p,X}$ or (b) the completion $\hat{\mathcal{O}}_{p,X}$ at a closed point $p$ on a normal surface $X$ over $k$ [1] or [6].
(3) The quotient field of a ring obtained by (a) henselizing or (b) completing an affine surface over $k$ along an integral curve [4].

In fact in each of these three examples, each division algebra $D$ is split by a cyclic extension $K(\alpha^{1/n})$ for some $\alpha \in K$ and $n = \mathrm{exponent}(D)$.

The purpose of this paper is to add to the list of Example 1 another class of fields satisfying the exponent = degree property.

Let $\mathcal{O}_{p,X}$ be the local ring at a regular point $p$ on an algebraic curve $X$ over the field $k$. Then $\mathcal{O}_{p,X}$ is a local principal ideal domain, hence a discrete valuation ring. The residue field of $\mathcal{O}_{p,X}$ is $k$. Let $R = \mathcal{O}^h_{p,X}$ be the henselization of $\mathcal{O}_{p,X}$. Consider an affine algebraic curve $C$ over $R$. Following [9], $C$ is an affine scheme together with a structure morphism $\pi : C \to \operatorname{Spec} R$ such that $\pi$ is flat and of finite type, the fibers of $\pi$ are algebraic curves, and $C$ is connected. Then $\pi$ has 2 fibers. The closed fiber $\pi : C_0 \to x_0$ over the closed point $x_0$ of $\operatorname{Spec} R$ is an algebraic curve over $k$. The open fiber $\pi : C_\eta \to \eta$ over the open point of $\operatorname{Spec} R$ is an algebraic curve over the quotient field of $R$. Assume that $C_\eta$ is integral, with

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$K = K(C_\eta)$ the field of fractions. Our main result is that $K$ has the exponent = degree property for division algebras.

**Theorem 2.** *Let $R$, $C$ and $K$ be as above and let $D$ be a central finite dimensional $K$-division algebra with* $\text{exponent}(D) = n$. *Then $D$ is a cyclic algebra of degree $n$.*

*Proof.* The proof is in the flavor of those used by [6] and [4].

Since $C$ is $\operatorname{Spec} S$ for an algebra $S$ of finite type over $R$, we can assume $C$ is a closed subscheme of affine space $\mathbb{A}^m_R$ over $R$. Without changing $K$ we can replace $C$ with a projective completion over $R$. If necessary, we can also desingularize $C$. Therefore assume that $\pi : C \to \operatorname{Spec} R$ is proper, that the open fiber is a nonsingular integral curve $C_\eta$ over the quotient field of $R$. By Embedded Resolution of Curves in Surfaces [8, p. 391], we can assume that the closed fiber $(C_0)_{red}$ is a divisor over $k$ with normal crossings. That is, write the reduced closed fiber $(C_0)_{red}$ as a union $C_1 \cup C_2 \cup \cdots \cup C_s$ of irreducible curves. By the normal crossing hypothesis we assume each component $C_j$ is a nonsingular curve and that $(C_0)_{red}$ has at most ordinary double points as singularities.

Let $L/K$ be a finite extension of fields and $Y \to C$ the integral closure of $C$ in $L$. Let $f : Y' \to Y$ be any desingularization of $Y$. That is, $Y'$ is nonsingular and $f$ is a proper birational morphism. There is a complex

$$(1) \quad 0 \to \mathrm{B}(Y') \to \mathrm{B}(L) \xrightarrow{a} \bigoplus_{\Delta} H^1(K(\Delta), \mathbb{Q}/\mathbb{Z}) \xrightarrow{r} \bigoplus_{P} \mu(-1) \xrightarrow{s} H^4(Y', \mu) \to 0$$

which is exact except possibly at the term $\bigoplus H^1(K(\Delta), \mathbb{Q}/\mathbb{Z})$. The first summation is over all irreducible curves $\Delta \subseteq Y'$, the second over all closed points $P \in Y'$. This follows by combining sequences (3.1) and (3.2) of [2]. If $H^3(Y', \mu) = 0$, (1) is exact. The first two groups in (1) are the Brauer groups respectively, of $Y'$ and $L$. The map $a$ "measures the ramification" of a division algebra $\Lambda$ over $L$. The ramification divisor of $\Lambda$ is the set of divisors $\Delta$ where $a[\Lambda]$ is nontrivial. The group $H^1(K(\Delta), \mathbb{Q}/\mathbb{Z})$ classifies the cyclic Galois extensions of $K(\Delta)$ the function field of $\Delta$. The map $r$ measures the ramification of cyclic extensions of $K(\Delta)$. Here $\mu(-1) = \bigcup_n \operatorname{Hom}(\mu_n, \mathbb{Q}/\mathbb{Z})$. Let $D$ be a central $K$-division algebra and $D_L = D \otimes L$, the restriction of $D$ to $L$. We say that $L$ splits the ramification of $D$ on $C$ if there exists a desingularization $f : Y' \to Y$ such that the class of $D_L$ in the Brauer group $\mathrm{B}(L)$ is in the image of the Brauer group $\mathrm{B}(Y')$ of $Y'$.

We proceed as in the proof of [4, Cor. 5]. Since $R$ is a direct limit of étale neighborhoods of $(X, p)$, $C$ is of finite type over $R$ and $D$ is a finite $K$-algebra, we can find an étale neighborhood $(U, p)$ of $(X, p)$, and a nonsingular algebraic surface $C_1$ satisfying the following.

(1) There is a proper morphism $C_1 \to U$.
(2) $C = C_1 \times_U \operatorname{Spec} R$
(3) If $K_1$ is the function field of $C_1$, then there is a central simple algebra $D_1$ over $K_1$ such that $D = D_1 \otimes_{K_1} K$.

It was shown in the text immediately preceding Theorem 1.6 of [6] and again in [4, Prop. 3] that there exists a surface $C_2$ and a proper birational morphism $C_2 \to C_1$ and a cyclic field extension $L_1/K_1$ of degree $n$ such that $L_1$ splits the ramification of $D_1$ on $C_2$. Furthermore, if $Y_1$ is the integral closure of $C_2$ in $L_1$, then $Y_1$ has only rational singularities. Let $L$ denote the field $KL_1$. Then $L/K$ is

cyclic of degree $n$. Set $C' = C_2 \times_U \operatorname{Spec} R$. Let $Y$ denote the integral closure of $C'$ in $L$. By the construction of $C_2$, $Y$ has at most rational singularities and $D \otimes L$ is unramified at each prime divisor on $Y$. There is a desingularization $Y' \to Y$ and $D \otimes L$ is unramified on $Y'$. That is, $D \otimes L$ represents a class in the image of the Brauer group of $Y'$. It therefore suffices to show that $Y'$ has trivial Brauer group. But $Y' \to R$ satisfies the hypothesis of Theorem 3 (which is stated and proved below), so $D \otimes L$ is split. It follows that $D$ is a cyclic algebra of degree $n$. □

**Theorem 3.** *Let $R$, $C$ and $K$ be as in Theorem 2. Assume moreover that $\pi : C \to \operatorname{Spec} R$ is proper, $C$ is regular, that the fibers of $\pi$ are one dimensional, and that the closed fiber of $\pi$ is a curve over $k$ with normal crossings. Then $H^q(C, \mu) = 0$ for all $q \geq 3$ and $H^q(C, \mathbb{G}_m) = 0$ for all $q \geq 2$.*

*Proof.* Fix an integer $n \geq 2$. By proper base change $H^q(C, \mu_n) \cong H^q(C_0, \mu_n)$ for all $q \geq 1$, where $C_0$ is the closed fiber of $\pi$ (i.e. $C_0 = C \times_R x_0$ where $x_0$ is the closed point of $\operatorname{Spec} R$). Since $C_0$ is a curve over $x_0 = \operatorname{Spec} k$, $H^q(C_0, \mu_n) = 0$ for $q \geq 3$. Taking the direct limit over all $n$ gives $H^q(C_0, \mu) = 0$ for $q \geq 3$. The sequence of sheaves for the étale topology on $C$

$$\begin{array}{ccccccccc} & & & & x & \longrightarrow & x^n & & \\ 1 & \longrightarrow & \mu_n & \longrightarrow & \mathbb{G}_m & \longrightarrow & \mathbb{G}_m & \longrightarrow & 1 \end{array} \tag{2}$$

is exact by Kummer theory. The associated long exact sequence

$$\cdots \to H^q(C, \mu_n) \to H^q(C, \mathbb{G}_m) \xrightarrow{n} H^q(C, \mathbb{G}_m) \to \dots \tag{3}$$

shows that multiplication by $n$ is an isomorphism on $H^q(C, \mathbb{G}_m)$ for $q \geq 3$. Since $C$ is regular, by [7, II, p. 71] $H^q(C, \mathbb{G}_m)$ is a torsion group for all $q \geq 2$. Therefore $H^q(C, \mathbb{G}_m) = 0$ for all $q \geq 3$. Now we check that the Brauer group of $C$, $\mathrm{B}(C) = H^2(C, \mathbb{G}_m)$, is trivial. We use the Kummer sequence (3) for $q = 2$

$$0 \to \frac{\operatorname{Pic} C}{n \operatorname{Pic} C} \to H^2(C, \mu_n) \to {}_n\mathrm{B}(C) \to 0 \tag{4}$$

together with the fact that $H^2(C, \mu_n) \cong H^2(C_0, \mu_n)$. We assume $C_0$ is reduced, since $H^2(C_0, \mu_n) \cong H^2((C_0)_{red}, \mu_n)$. Write $C_0 = C_1 \cup C_2 \cup \cdots \cup C_s$ as a union of nonsingular irreducible curves. We assume each component $C_j$ is a nonsingular curve by the normal crossing hypothesis. By the Kummer sequence (4), the known description of $\operatorname{Pic} C_j$ and the fact that $\mathrm{B}(C_j) = 0$ (see for example [10, pp. 175–176]), it follows that $H^2(C_j, \mu_n) \cong \mathbb{Z}/n$ is generated by the class of any prime divisor on $C_j$. Now $H^2(C_0, \mu_n) \cong \coprod_{j=1}^{s} H^2(C_j, \mu_n)$ by Lemma 4 below. It suffices to show that for each $j = 1, \dots, s$, there exists a divisor $D_j$ on $C$ such that $D_j \cap C_j$ is a prime divisor on $C_j$ and $D \cap C_i = \emptyset$ if $i \neq j$. Let $P_0$ be a prime divisor on $C_j$ not in the singular locus of $C_0$ and pick any prime divisor $D_j$ on $C$ such that the intersection multiplicity of $D_j$ and $C_j$ at the closed point $P_0$ is 1. This is possible since $C$ and $C_j$ are both regular at $P_0$. The problem that one must worry about is the possibility that $D_j$ intersects $C_0$ at some other point. We check that this cannot happen since $D_j$ is integral and $R$ is henselian. Now $\iota : D_j \hookrightarrow C$ is a closed immersion, hence is proper. Furthermore $D_j$ does not contain any component $C_i$ of $C_0$. Also $D_j$ is closed, so $D_j$ does not contain $C_\eta$. Consider the composite $f = \pi \circ \iota : D_j \to \operatorname{Spec} R$. Since the fibers of $\pi$ are of dimension 1, $f^{-1}(x)$ is finite

for each $x$ in $\operatorname{Spec} R$. So $f$ is quasi-finite. Since $f$ is a composite of proper morphisms, $f$ is proper. But a proper quasi-finite morphism is finite [10, p. 6] so $D_j$ is finite over $R$. Since $R$ is henselian, any connected component of $D_j \times_R x_0$ gives rise to a connected component of $D_j$. But $D_j$ is integral, hence $D_j \times_R x_0 = P_0$. So ${}_n\mathrm{B}(C) = 0$ for each $n > 1$. □

**Lemma 4.** *Let $C$ be an algebraic curve over $k$ embedded with normal crossings on a surface $S$. If the irreducible components of $C$ are $C_1, \ldots, C_s$, then $H^2(C, \mu_n) \cong \coprod_{i=1}^{n} H^2(C_i, \mu_n)$.*

*Proof.* Let $D = C_1 \coprod \cdots \coprod C_s$ denote the disjoint union of the curves $C_1, \ldots, C_s$. There is an obvious finite projection $\pi : D \to C$. Let $\sigma C$ denote the singular locus of $C$ and $\sigma D = \pi^{-1}(\sigma C)$ those points on $D$ lying over $\sigma C$. Since $C$ has only nodal singularities, the map $\pi : \sigma D \to \sigma C$ is 2-to-1. Let $P$ be an element of $\sigma C$ and consider the cohomology with supports in $P$, $H^2_P(C, \mu_n)$. The strictly local ring $\mathcal{O}^h_{C,P}$ is henselian with algebraically closed residue field $k$. Let $U$ denote $\operatorname{Spec} \mathcal{O}^h_{C,P}$ and let $P^h$ denote the closed point of $U$. By excision [10, Cor. 1.28, p. 93] $H^2_P(C, \mu_n) \cong H^2_{P^h}(U, \mu_n)$. The long exact sequence for $P^h \subseteq U$ is

$$\cdots \to H^1(U, \mu_n) \to H^1(U - P^h, \mu_n) \to H^2_{P^h}(U, \mu_n) \to H^2(U, \mu_n) \to \ldots. \tag{5}$$

The curve $U$ consists of 2 nonsingular henselian curves $U_1$, $U_2$ crossing at the closed point $P^h$. Each curve $U_i$ is the prime spectrum of a henselian discrete valuation ring with residue field $k$. So $H^i(U, \mu_n) = 0$ for $i > 0$ and $H^1(U - P^h, \mu_n) = H^1(U_1 - P^h, \mu_n) \oplus H^1(U_2 - P^h, \mu_n) \cong \mathbb{Z}/n \oplus \mathbb{Z}/n$. So equation (5) and excision show that $H^2_P(C, \mu_n) \cong (\mathbb{Z}/n)^{(2)}$. If $Q$ is an element of $\sigma D$, then the argument above also shows that $H^2_Q(D, \mu_n) \cong \mathbb{Z}/n$. Since $\sigma C$ decomposes into a finite number of points $P$, it follows that $H^2_{\sigma C}(C, \mu_n)$ decomposes into the direct sum $\coprod_{P \in \sigma C} H^2_P(C, \mu_n)$ and similarly for $\sigma D$. The long exact sequences of cohomology with supports in $\sigma C$ and $\sigma D$ combined with the maps induced by $\pi$ yield the commutative diagram below.

(6)

$$\begin{array}{ccccccc}
H^1(C - \sigma C, \mu_n) & \longrightarrow & H^2_{\sigma C}(C, \mu_n) & \longrightarrow & H^2(C, \mu_n) & \longrightarrow & H^2(C - \sigma C, \mu_n) \\
\downarrow \alpha & & \downarrow \beta & & \downarrow \gamma & & \downarrow \delta \\
H^1(D - \sigma D, \mu_n) & \longrightarrow & H^2_{\sigma D}(D, \mu_n) & \longrightarrow & H^2(D, \mu_n) & \longrightarrow & H^2(D - \sigma D, \mu_n)
\end{array}$$

Because $\pi : C - \sigma C \xrightarrow{\cong} D - \sigma D$, $\alpha$ and $\delta$ are isomorphisms. The map $\beta$ is an isomorphism by the above computations. Therefore $\gamma$ is an isomorphism. Because $D$ is a disjoint union, the lemma follows. □

**Corollary 5.** *Let $R$, $C$ and $K$ be as in Theorem 3. The sequence*

$$0 \to \mathrm{B}(K) \xrightarrow{a} \bigoplus_{\Delta} H^1(K(\Delta), \mathbb{Q}/\mathbb{Z}) \xrightarrow{r} \bigoplus_{P} \mu(-1) \to 0$$

*is exact where the first summation is over all irreducible curves $\Delta \subseteq C$, the second over all closed points $P \in C$.*

*Proof.* Follows immediately from (1) and Theorem 3. □

For any discrete valuation ring $R$ with perfect residue field $k$ and field of fractions $K$, for each $q \geq 0$ the natural map $H^q(R, \mathbb{G}_m) \to H^q(k, \mathbb{G}_m)$ is an isomorphism [7, III, p. 93]. For $q = 2$ this is the theorem of Azumaya. There is a split-exact sequence

$$0 \to H^q(k, \mathbb{G}_m) \to H^q(K, \mathbb{G}_m) \to H^{q-1}(k, \mathbb{G}_m) \to 0$$

for each $q \geq 2$ [7, III, p. 93 and p. 188]. In particular, for $q = 2$, it follows that the Brauer group of $K$ decomposes into a direct sum of $\mathrm{B}(k)$ and $H^1(k, \mathbb{Q}/\mathbb{Z})$. The group $H^1(k, \mathbb{Q}/\mathbb{Z})$ parametrizes the unramified cyclic Galois extensions of $K$. Various other results along these same lines are derived in [3] and [11]. We arrive at similar results for the Brauer group of a curve over $K$.

**Theorem 6.** *Let $R$ be a strictly local ring which is the henselization of a local ring $\mathcal{O}_{p,X}$ at a closed point $p$ on a smooth curve $X$ over $k$. Let $\pi : C \to \operatorname{Spec} R$ be proper and smooth of relative dimension 1. Let $\eta$ be the generic point of $\operatorname{Spec} R$ and $C_\eta = C \times_R \eta$. Let $x_0$ be the closed point of $\operatorname{Spec} R$ and $C_0 = C \times_R x_0$. Then the Brauer group $\mathrm{B}(C_\eta)$ is isomorphic to $H^1(C_0, \mathbb{Q}/\mathbb{Z})$. Every Azumaya algebra over $C_\eta$ of exponent $n$ is split by a cyclic Galois extension of degree $n$ which descends to an unramified extension of $C$.*

*Proof.* Since $\operatorname{Spec} R = \{x_0\} \cup \{\eta\}$ with $\{x_0\}$ closed and $\{\eta\}$ open, we have $C = C_0 \cup C_\eta$ with $C_0$ closed and $C_\eta$ open. The long exact sequence of cohomology with supports in $C_0$ and coefficients in $\mathbb{G}_m$ is

$$\begin{aligned}(7)\quad H^0(C, \mathbb{G}_m) \to H^0(C_\eta, \mathbb{G}_m) \to H^1_{C_0}(C, \mathbb{G}_m) \to \\ H^1(C, \mathbb{G}_m) \to H^1(C_\eta, \mathbb{G}_m) \to H^2_{C_0}(C, \mathbb{G}_m) \to \\ H^2(C, \mathbb{G}_m) \to H^2(C_\eta, \mathbb{G}_m) \to H^3_{C_0}(C, \mathbb{G}_m) \to \dots\end{aligned}$$

If $t$ is a local parameter for $R$, then $\dfrac{H^0(C_\eta, \mathbb{G}_m)}{H^0(C, \mathbb{G}_m)} = \langle t \rangle$ and $t$ vanishes with order 1 along $C_0$. Since $C_0$ is a principal divisor, $\operatorname{Pic} C \cong \operatorname{Pic} C_\eta$. From Kummer theory the diagram

(8)

$$\begin{array}{ccccccccc} 0 & \longrightarrow & H^0(C_\eta, \mathbb{G}_m) \otimes \mathbb{Z}/n & \longrightarrow & H^1(C_\eta, \mu_n) & \longrightarrow & {}_n \operatorname{Pic} C_\eta & \longrightarrow & 0 \\ & & \uparrow & & \uparrow & & \uparrow \cong & & \\ 0 & \longrightarrow & H^0(C, \mathbb{G}_m) \otimes \mathbb{Z}/n & \longrightarrow & H^1(C, \mu_n) & \longrightarrow & {}_n \operatorname{Pic} C & \longrightarrow & 0 \end{array}$$

commutes and has exact rows. Since $H^0(C_\eta, \mathbb{G}_m) \otimes \mathbb{Z}/n \cong \langle t \rangle / \langle t^n \rangle$ and $H^0(C, \mathbb{G}_m) \otimes \mathbb{Z}/n = 0$, we see from (8) that $H^1(C_\eta, \mu_n) \cong \langle t \rangle / \langle t^n \rangle \times H^1(C, \mu_n)$ and by proper base change $H^1(C, \mu_n) \cong H^1(C_0, \mu_n)$. From Theorem 3 we have $H^q(C, \mathbb{G}_m) = 0$ for all $q \geq 2$. From (7) it follows that $H^q(C_\eta, \mathbb{G}_m) \cong H^{q+1}_{C_0}(C, \mathbb{G}_m)$ for all $q \geq 2$. Since $\operatorname{Spec} R$ is a direct limit of étale neighborhoods $U \hookrightarrow X$ of the closed point $p \hookrightarrow X$, the morphism $\pi : C \to \operatorname{Spec} R$ descends to a proper smooth morphism $C' \to U$ with a closed fiber $C'_0 = C' \times_U p \cong C_0$. By cohomological purity (see [5]) $H^q_{C'_0}(C', \mathbb{G}_m) \cong H^{q-2}(C_0, \mathbb{Q}/\mathbb{Z})$ for all $q \geq 3$. Taking the limit over all such $U \hookrightarrow X$ yields $H^q_{C_0}(C, \mathbb{G}_m) \cong H^{q-2}(C_0, \mathbb{Q}/\mathbb{Z})$ for all $q \geq 3$. Therefore $H^2(C_\eta, \mathbb{G}_m) \cong H^1(C_0, \mathbb{Q}/\mathbb{Z})$, $H^3(C_\eta, \mathbb{G}_m) \cong H^2(C_0, \mathbb{Q}/\mathbb{Z}) \cong \mu$ and $H^q(C_\eta, \mathbb{G}_m) \cong H^{q-1}(C_0, \mathbb{Q}/\mathbb{Z}) = 0$ for all $q \geq 4$. The isomorphism ${}_nH^2(C_\eta, \mathbb{G}_m) \cong H^1(C_0, \mathbb{Z}/n)$

is the Gysin map [10, p. 244]. Given any $\lambda$ in $H^1(C, \mathbb{Z}/n)$ there is a corresponding $\lambda_0$ in $H^1(C_0, \mathbb{Z}/n)$. Let $\lambda_\eta$ denote the image of $\lambda$ in the group $H^1(C_\eta, \mathbb{Z}/n)$ under the natural map. Let $\lambda_\eta$ also denote the corresponding cyclic Galois cover of $C_\eta$ with group $\langle\sigma\rangle$. Using the cyclic Galois cover $\lambda_\eta$ and the trivial factor set $t$ we form a cyclic crossed product algebra $\Delta(\lambda) = (\lambda_\eta/C_\eta, \sigma, t)$ which represents a class in ${}_n\,\mathrm{B}(C_\eta)$. Consider the ramification divisor of $\Delta(\lambda)$ on $C$. Along the divisor $C_0$ of $C$, the ramification of $\Delta(\lambda)$ is the element $\lambda_0$ of the group $H^1(C_0, \mathbb{Z}/n)$. Therefore, the correspondence $\lambda \mapsto \Delta(\lambda)$ induces an isomorphism $H^1(C, \mathbb{Z}/n) \cong {}_n\,\mathrm{B}(C_\eta)$. Every Azumaya algebra over $C_\eta$ whose Brauer class is annihilated by $n$ is Brauer equivalent to a cyclic crossed product of the form $(\lambda_\eta/C_\eta, \sigma, t)$, hence is split by $\lambda_\eta$ for some $\lambda$. □

## References


[1] M. Artin, *Two dimensional orders of finite representation type*, Manuscripta Math., **58** (1987), pp. 445–471.

[2] M. Artin and D. Mumford, *Some elementary examples of unirational varieties which are not rational*, Proc. London Math. Soc. (3), **25** (1972), pp. 75–95.

[3] M. Auslander and A. Brumer, *Brauer groups of discrete valuation rings*, Nederl. Akad. Wetensch. Proc. Ser. A, **71** (1968), pp. 286–296.

[4] T. J. Ford, *Division algebras over nonlocal henselian surfaces*, Pacific J. Math., **147** (1991), pp. 301–310.

[5] ———, *On the Brauer group of a localization*, J. Algebra, **147** (1992), pp. 365–378.

[6] T. J. Ford and D. Saltman, *Division algebras over henselian surfaces*, Israel Mathematical Conference Proceedings, **1** (1989), pp. 320–336.

[7] A. Grothendieck, *Le groupe de Brauer I, II, III*, in Dix Exposés sur la Cohomologie des Schémas, North Holland, Amsterdam, 1968, pp. 46–188.

[8] R. Hartshorne, *Algebraic Geometry*, vol. 52 of Graduate Texts in Mathematics, Springer-Verlag, New York/Berlin, 1977.

[9] S. Lichtenbaum, *Curves over discrete valuation rings*, Amer. J. Math., **90** (1968), pp. 380–405.

[10] J. Milne, *Etale Cohomology*, vol. 33 of Princeton Mathematical Series, Princeton University Press, Princeton, N.J., 1980.

[11] S. Yuan, *On the Brauer groups of local fields*, Ann. of Math. (2), **82** (1965), pp. 434–444.



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