ERRATA TO "SEPARABLE ALGEBRAS" BY T. J. FORD GRADUATE STUDIES IN MATHEMATICS, VOL. 183, 2017 February 16, 2024

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(11/09/2023) I am grateful to Philippe Gille and M. Bruneaux for pointing this out to me. In the exact sequence of Proposition 10.4.9, the map ρ is not necessarily one-to-one. In the statement of Proposition 10.4.9, change the exact sequence of pointed sets to:

$$\check{\mathrm{H}}^1_{\mathrm{et}}(R,\mathbb{G}_m) \xrightarrow{\rho} \check{\mathrm{H}}^1_{\mathrm{et}}(R,\mathrm{GL}_n) \xrightarrow{\chi} \check{\mathrm{H}}^1_{\mathrm{et}}(R,\mathrm{PGL}_n) \xrightarrow{\partial} \check{\mathrm{H}}^2_{\mathrm{et}}(R,\mathbb{G}_m)$$

In the proof of Proposition 10.4.9, delete: "The proof that ρ is one-to-one is left to the reader."

(07/31/2023) I am grateful to Erhard Neher for bringing to my attention the following problems that appear in Section 14.1 and for offering the suggestions for alternate proofs of Theorems 14.1.8 (3) and 14.1.9 (3) which are given below.

A goal on my to-do list is to correct Theorem 14.1.5 and rewrite the proofs of Theorems 14.1.8 (3) and 14.1.9 (3). Until then, Theorem 14.1.5 (3) should be assumed false.

It seems that the second part of [KO75, Theorem 3.2 (2)], which is part (3) of Theorem 14.1.5, is not correct. M. Ojanguren mentions this in his review of [Ver88] for Math Reviews.

If Theorem 14.1.5 (3) is not correct, this has an impact on the proof of Theorem 14.1.8 (3). However, we only need this for invertible modules, which has been proved in [Gro61, EGA II, (6.5.2.4)]. As pointed out in [KO75, Example 6.1], our norm of an invertible module agrees with that of [Gro61, EGA II].

The same problem exists with the proof of Theorem 14.1.9 (3), which is also based on Theorem 14.1.5 (3). The result however is correct, as is shown in [Fer98, Remark 7.3.6], or in [Sal99, Corollary 8.2].

(07/28/2023) I am grateful to Erhard Neher for pointing this out to me. On p. 581, line 4, change: Section 13.8.1 to: Section 7.8.1.

(07/28/2023) On p. 582, in Theorem 14.1.17, the cohomology groups in the commutative diagram should both be changed to H^1 groups. That is, change the commutative diagram to:

$$\begin{tikzcd} \mathrm{B}(L'/S) \arrow[r, "\alpha_5"] \arrow[d, "\mathrm{Cor}_R^S"'] & \mathrm{H}^1(G,\mathrm{Pic}(L')) \arrow[d, "\mathrm{Cor}_L^{L'}"] \\ \mathrm{B}(L/R) \arrow[r, "\alpha_5"] & \mathrm{H}^1(G,\mathrm{Pic}(L)) \end{tikzcd}$$

(02/20/2023) On p. 261, in the penultimate sentence of the proof of Theorem 7.5.4, change: $m > 0$ to $[D^* : F^*] > 0$.

(10/11/2021) On p. 267, in Section 7.7: In the opening paragraph, change: "$R_0 \subseteq R$, where R_0 is a finitely generated $\mathbb{Z}$-algebra." to: "$R_0 \subseteq R$, where R_0 is a finitely generated subring of R. If R is commutative, then R_0 can be taken to be a finitely generated $\mathbb{Z}$-algebra, hence can be taken to be noetherian"

On p. 267, in Proposition 7.7.1 change: "Then there is a noetherian subring $R_0 \subseteq R$ (in fact R_0 can be taken to be a finitely generated $\mathbb{Z}$-algebra)" to: "Then there is a finitely generated subring $R_0 \subseteq R$ (if R is commutative, then R_0 can be taken to be a finitely generated $\mathbb{Z}$-algebra, hence can be taken to be noetherian)"

(09/13/2021) On p. 300, line 3, in the proof of (2) implies (1) of Theorem 8.1.24, change: Corollary 1.3.19 to Corollary 1.3.18.

(06/26/2021) On p. 371, in the proof of (3) implies (4) of Theorem 10.4.1, change: S_0 to B_0.

(07/03/2020) I wish to thank Nguyen Xuan Bach for pointing this out to me. On p. 269, in the proof of Proposition 7.7.2, there is a gap because the top rows of Diagrams (7.9) and (7.10) are not exact. To correct this error, replace the entire first paragraph of the proof with this:

The finitely generated projective R-module A is a direct summand of R^n, for some $n \geq 1$. Therefore, there is an idempotent a in $\operatorname{Hom}_R(R^n, R^n)$ and an exact sequence of R-modules

$$R^n \xrightarrow{a} R^n \xrightarrow{c} A \to 0. \tag{7.8}$$

Let $\{v_1, \ldots, v_n\}$ be the standard basis for R^n. Without loss of generality, assume the image of v_1 under c is 1, the multiplicative identity of A. As in Proposition 7.7.1, the commutative diagram

$$\begin{tikzcd}
(R^n \otimes R^n) \oplus (R^n \otimes_R R^n) \arrow[r, "a\otimes 1 + 1 \otimes a"] \arrow[d, "\psi"] & R^n \otimes_R R^n \arrow[r, "c \otimes c"] \arrow[d, "\phi"] & A \otimes_R A \arrow[r] \arrow[d, "\mu"] & 0 \\
R^n \arrow[r, "a"] & R^n \arrow[r, "c"] & A \arrow[r] & 0
\end{tikzcd} \tag{7.9}$$

results from combining (7.8) with the multiplication map μ. The top row of (7.9) is exact, by Lemma 5.2.2. Let R_0 be the subring of R generated by the entries in the matrices of a, ψ, and ϕ with respect to the standard basis for R^n. Then the matrices descend to define R_0-module homomorphisms

a_0, ψ_0, and ϕ_0. Define A_0 to be the cokernel of a_0. We get a commutative diagram

(7.10)

$$\begin{tikzcd}
(R_0^n \otimes_{R_0} R_0^n) \oplus (R_0^n \otimes_{R_0} R_0^n) \arrow[r, "a_0 \otimes 1 + 1 \otimes a_0"] \arrow[d, "\psi_0"] & R_0^n \otimes_{R_0} R_0^n \arrow[r] \arrow[d, "\phi_0"] & A_0 \otimes_{R_0} A_0 \arrow[r] \arrow[d, "\mu_0"] & 0 \\
R_0^n \arrow[r, "a_0"] & R_0^n \arrow[r] & A_0 \arrow[r] & 0
\end{tikzcd}$$

where the top row is exact, by Lemma 5.2.2 and μ_0 is induced by the rest of the diagram. The proof of Proposition 7.7.1 shows that A_0 is a finitely generated projective R_0-module, $A = A_0 \otimes_{R_0} R$, and $A_0 \subseteq A$. Since (7.10) commutes, A_0 is an R_0-subalgebra of A. This proves (1) and (2).

(03/13/2018) On p. 256, in the statement of Theorem 7.4.3,
change: $\operatorname{Rank}_{R_\mathfrak{p}}(A_\mathfrak{p})$ to: $\operatorname{Rank}_{R_\mathfrak{p}}(B_\mathfrak{p})$.

(11/4/2017) On p. 62, in Lemma 2.2.7 part (2), add a period after the displayed equation.

(11/4/2017) On p. 19, in Section 1.2.3, line 3,
change: ...purpose of this section is to proof ...
to: ...purpose of this section is to prove ....

(9/26/2017) On p. 614, in Exercise 14.3.19,
change: $R = k[x, y](xy - 1)$ to: $R = k[x, y]/(xy - 1)$
change: $R = k[x, y](x^2y - 1)$ to: $R = k[x, y]/(x^2y - 1)$
change: $R = k[x, y](y^2 - x^3 + x)$ to: $R = k[x, y]/(y^2 - x^3 + x)$
change: $R = k[x, z](z - x^3 + xz^2)$ to: $R = k[x, z]/(z - x^3 + xz^2)$

(7/5/2017) On p. 602, in Section 14.3.3 in the third paragraph (the paragraph that defines the ring R), between sentences one and two, add: Assume $p(x)$ is not a square.

(7/5/2017) On p. 595, in the proof of Theorem 14.2.12, in the last paragraph of Step 1, change: ∂ to: ∂_1.

References

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