ON THE BRAUER GROUP AND THE CUP PRODUCT MAP

TIMOTHY J. FORD
Department of Mathematics
Florida Atlantic University
Boca Raton, Florida 33431

ABSTRACT. This article is concerned with the cup product map

$$\mu : \mathrm{H}^1(X,\mathbb{Z}/n)\otimes\mathrm{H}^1(X,\mathbb{Z}/n)\to {}_n\mathrm{B}(X)\ .$$

Under certain conditions we describe the image and kernel of μ for the spectrum of $k[x_1,\ldots,x_\nu,f^{-1}]$ and for a fiber product space.

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Throughout X will be a connected scheme over $\mathbb{Z}[1/n][\omega]$ where $n > 1$ is an integer and ω is a primitive n-th root of unity. We denote by $\mathrm{B}(X)$ the Brauer group of X and by $\mathrm{B}'(X)$ the cohomological Brauer group of X [13]. For any abelian group A we let ${}_nA$ denote the subgroup of A annihilated by n. All cohomology and sheaves are for the étale topology. Let $\mathbb{G}_m$ denote the sheaf of units on X and μ_n the

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sheaf of n-th roots of unity. The sequence

$$1 \to \mu_n \to \mathbb{G}_m \xrightarrow{n} \mathbb{G}_m \to 1 \tag{1}$$

is exact. Since $\Gamma(X, \mathbb{G}_m)$ contains ω, μ_n is (non-canonically) isomorphic to the constant sheaf $\mathbb{Z}/n$. The long exact sequence of cohomology associated to (1) is

$$\begin{aligned}(2)\quad 1 \to \mu_n \to \Gamma(X, \mathbb{G}_m) \xrightarrow{n} \Gamma(X, \mathbb{G}_m) \to \mathrm{H}^1(X, \mathbb{Z}/n) \to \operatorname{Pic} X \xrightarrow{n} \operatorname{Pic} X \\ \to \mathrm{H}^2(X, \mathbb{Z}/n) \to \mathrm{B}'(X) \xrightarrow{n} \mathrm{B}'(X) \to \dots\end{aligned}$$

where we have identified $\operatorname{Pic} X = \mathrm{H}^1(X, \mathbb{G}_m)$ and $\mathrm{B}'(X) = \operatorname{tors}\big(\mathrm{H}^2(X, \mathbb{G}_m)\big)$. If X is affine, it is known that $\mathrm{B}(X) = \mathrm{B}'(X)$ under the natural map $\mathrm{B}(X) \to \mathrm{H}^2(X, \mathbb{G}_m)$ [11], [17]. The cup product map [21, V.1.17]

$$\mathrm{H}^1(X, \mathbb{Z}/n) \otimes \mathrm{H}^1(X, \mathbb{Z}/n) \to \mathrm{H}^2(X, \mathbb{Z}/n)$$

followed by the homomorphism

$$\mathrm{H}^2(X, \mathbb{Z}/n) \to {}_n\mathrm{B}'(X)$$

defines a homomorphism

$$\mu : \mathrm{H}^1(X, \mathbb{Z}/n) \otimes \mathrm{H}^1(X, Z/n) \to {}_n\mathrm{B}'(X) \tag{3}$$

which will also be called cup product.

This article is concerned with the study of the map μ. If X is the spectrum of a field k this problem has been completely solved by Merkurjev [19], [22] if $n = 2$ and by Merkurjev and Suslin [20] for all $n > 1$. For $\operatorname{Spec} k$, μ is always surjective and $\ker \mu$ is the Steinberg relation group of k. In [3] L. Childs shows that if R is the ring of algebraic integers in a number field, then ${}_n\mathrm{B}(R)$ is not always generated by $\operatorname{im}\mu$.

The group $\mathrm{H}^1(X,\mathbb{Z}/n)$ classifies Galois covers of X with group $\mathbb{Z}/n$. It is known that μ corresponds to taking the smash product of two cyclic Galois covers of X [12]. Since the smash product of cyclic Galois extensions is an Azumaya algebra, $\operatorname{im}\mu \subseteq {}_n\mathrm{B}(X)$. When $n=2$ it is shown in [7] that μ is intimately connected to the group structure of the Brauer-Wall group $\mathrm{BW}(X)$ and the Brauer-Long group $\mathrm{BD}(X,\mathbb{Z}/2)$. To compute $\mathrm{BD}(X,\mathbb{Z}/2)$ it suffices to compute $\mathrm{B}(X)$, $\mathrm{H}^1(X,\mathbb{Z}/2)$, and the cup product map μ.

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First we consider rings of the form $R=k[x_1,\dots,x_\nu,f^{-1}]$. If f factors into linear polynomials, Theorem 1 shows μ is onto and $\ker\mu$ is described. Examples 2 and 3 illustrate that this is not the case in general.

Let $Y_0,\dots,Y_m$ be distinct hyperplanes in $\mathbb{P}^\nu$, $\nu>1$. Let $Y=Y_0\cup\cdots\cup Y_m$. Let P denote the singular set of Y, $P=\{Y_i\cap Y_j \mid i\neq j\}$. Write $P=p_1\cup\cdots\cup p_s$ where the p_i are the irreducible components of P. Each p_i is a linear subvariety of $\mathbb{P}^\nu$ of codimension 2, hence is isomorphic to $\mathbb{P}^{\nu-2}$. Define a graph Γ associated to Y. The vertices of Γ are the hyperplanes $Y_0,\dots,Y_m$ and the varieties $p_1,\dots,p_s$. There is an edge connecting Y_i and p_j if and only if p_j is a subvariety of Y_i. The graph Γ is bipartite and connected. We orient Γ by taking the positive end of an edge E the Y_i and the negative end the p_j. Let e be the number of edges.

Theorem 1. [10, Theorem 1] *Let k be an algebraically closed field of characteristic p. Let $f_1,\dots,f_m$ be linear polynomials in $k[x_1,\dots,x_\nu]$ and*

$$R = k[x_1,\dots,x_\nu][f_1^{-1},\dots,f_m^{-1}] .$$

Let Y_0 be the hyperplane at infinity and $Y_1,\dots,Y_m$ the complete hyperplanes in $\mathbb{P}^\nu$ defined by $f_1,\dots,f_m$. Assume that the Y_i are distinct. Let $Y = Y_0 \cup \cdots \cup Y_m$ and Γ the graph of Y. Then modulo p-groups $\mathrm{B}(R) = \mathbb{Q}/\mathbb{Z}^{(r)}$ where $r = e - m - s$ is the rank of the cycle space of Γ. The cup product map

$$\mu : \mathrm{H}^1(R,\mathbb{Z}/n) \otimes \mathrm{H}^1(R,\mathbb{Z}/n) \to {}_n\mathrm{B}(R)$$

is surjective for all n relatively prime to p and $\ker\mu$ *is generated by*

$$\{f_i \otimes f_j | Y_i \cap Y_0 = Y_j \cap Y_0\} \bigcup \{(f_i \otimes f_j)(f_j \otimes f_t)(f_i \otimes f_t)^{-1} | Y_i \cap Y_t = Y_j \cap Y_t\} .$$

Example 2. Let $k = \mathbb{C}$ be the field of complex numbers. Choose four points in the affine plane over k not all on a conic of the form $y = ax^2 + bx + c$ and no three on a line. Choose four conics A, B, C, D each with equation of the form $y = ax^2 + bx + c$, each passing through exactly three of the above points, no two conics containing the same three points. Let $R = k[x,y][\alpha^{-1},\beta^{-1},\gamma^{-1},\delta^{-1}]$ where α, β, γ, δ are the polynomials in k[x,y] corresponding to A, B, C, D. In [10] it is shown that $\mathrm{H}^1(R,\mathbb{Z}/2) = (\mathbb{Z}/2)^{(4)}$, ${}_2\mathrm{B}(R) = (\mathbb{Z}/2)^{(8)}$ and $\operatorname{im}\mu \cong (\mathbb{Z}/2)^{(5)}$. Thus μ is not surjective.

Example 3. Let k be an algebraically closed field and n relatively prime to the characteristic of k. Let $f = x^n - y^{n-1}z$ and $R = k[x,y,z,f^{-1}]$. In [10] it is shown that $\mathrm{H}^1(R,\mathbb{Z}/n) \cong \mathbb{Z}/n$, $\mathrm{B}(R) \cong \mathbb{Z}/n$ and μ is the zero map.

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Now we consider fiber product spaces. In Example 4 we see that the Brauer group of a Laurent polynomial ring is generated by cup products and the Brauer group of the base ring. Corollary 6, a Künneth formula for the Brauer group, gives sufficient conditions for ${}_n\mathrm{B}'(X \times Y)$ to be generated by ${}_n\mathrm{B}'(X)$, ${}_n\mathrm{B}'(Y)$, and cup products.

Example 4. Suppose R is a $\mathbb{Z}[1/n][\omega]$-algebra and $\operatorname{Spec} R$ is connected. Let t be an indeterminate. In [9] it is shown

$$ {}_n\mathrm{B}\big(R[t,1/t]\big) \cong {}_n\mathrm{B}(R) \oplus \big(\mathrm{H}^1(R,\mathbb{Z}/n)/(C/nC)\big) \tag{4} $$

where $C = \operatorname{Pic} R[t,1/t]/\operatorname{Pic} R$. The homomorphism $\mathrm{H}^1(R,\mathbb{Z}/n) \to {}_n\mathrm{B}(R)$ is induced (non-canonically) by taking the smash product of a cyclic Galois extension L with the cyclic extension $R[t,1/t][t^{1/n}]$. Therefore ${}_n\mathrm{B}\big(R[t,1/t]\big)$ is generated by ${}_n\mathrm{B}(R)$ and $\operatorname{im}\mu$. If R contains an algebraically closed field, this is a special case of Corollary 6.

Theorem 5. [21, VI.8.25] *and* [4, Th. finitude, 1.11]. *Let X and Y be schemes of finite type over the separably closed field k. Let n be relatively prime to the characteristic of k. Let F and G be sheaves of $\mathbb{Z}/n$-modules on X and Y respectively. The Künneth map*

$$ R\Gamma(X,F) \otimes^L R\Gamma(Y,G) \to R\Gamma(X \times Y, F \boxtimes^L G) \tag{5} $$

is a quasi-isomorphism.

Corollary 6. *Let X and Y be connected schemes of finite type over the algebraically closed field k. Let $n > 1$ be relatively prime to the characteristic of k. Suppose* $\mathrm{H}^i(X,\mathbb{Z}/n)$ *is a free* $\mathbb{Z}/n$*-module for* $i > 0$*. Then the following sequences are exact, where C is defined by the first sequence.*

$$0 \to \operatorname{Pic} X \oplus \operatorname{Pic} Y \to \operatorname{Pic} X \times Y \to C \to 0 \tag{6}$$

$$0 \to C/nC \to \mathrm{H}^1(X,\mathbb{Z}/n) \otimes \mathrm{H}^1(Y,\mathbb{Z}/n) \xrightarrow{\mu} {}_n\mathrm{B}'(X \times Y) \to {}_n\mathrm{B}'(X) \oplus {}_n\mathrm{B}'(Y) \to 0 \tag{7}$$

Proof. Because $\mathrm{H}^i(X,\mathbb{Z}/n)$ is flat for $i > 0$, Theorem 5 gives

$$\mathrm{H}^2(X \times Y,\mathbb{Z}/n) \cong \bigoplus_{p+q=2} \mathrm{H}^p(X,\mathbb{Z}/n) \otimes \mathrm{H}^q(Y,\mathbb{Z}/n) \tag{8}$$

Since k is algebraically closed the natural projections $X \times Y \to X$, $X \times Y \to Y$ admit sections. Therefore, the natural maps $\mathrm{B}'(X) \oplus \mathrm{B}'(Y) \to \mathrm{B}'(X \times Y)$ and $\operatorname{Pic} X \oplus \operatorname{Pic} Y \to \operatorname{Pic} X \times Y$ split. Let C be defined by (6). Then $\operatorname{Pic} X \times Y \cong \operatorname{Pic} X \oplus \operatorname{Pic} Y \oplus C$ and $\operatorname{Pic}(X \times Y)/n\operatorname{Pic}(X \times Y) \cong \operatorname{Pic} X/n\operatorname{Pic} X \oplus \operatorname{Pic} Y/n\operatorname{Pic} Y \oplus C/nC$. Kummer theory (2) gives a commutative diagram

$$
\begin{array}{ccccccccc}
 & 0 & \longrightarrow & \frac{\operatorname{Pic} Y}{n \operatorname{Pic} Y} & \longrightarrow & \mathrm{H}^2(Y, \mathbb{Z}/n) & \longrightarrow & {}_n\mathrm{B}'(Y) & \longrightarrow & 0 \\
 & & & \downarrow & & \downarrow & & \downarrow & & \\
(9) & 0 & \longrightarrow & \frac{\operatorname{Pic} X\times Y}{n \operatorname{Pic} X \times Y} & \longrightarrow & \mathrm{H}^2(X \times Y, \mathbb{Z}/n) & \longrightarrow & {}_n\mathrm{B}'(X \times Y) & \longrightarrow & 0 \\
 & & & \uparrow & & \uparrow & & \uparrow & & \\
 & 0 & \longrightarrow & \frac{\operatorname{Pic} X}{n \operatorname{Pic} X} & \longrightarrow & \mathrm{H}^2(X, \mathbb{Z}/n) & \longrightarrow & {}_n\mathrm{B}'(X) & \longrightarrow & 0
\end{array}
$$

with split vertical arrows and exact rows. From (9) we have the exact sequence:

$$
(10) \quad 0 \to \frac{\operatorname{Pic}(X \times Y) \otimes \mathbb{Z}/n}{\left(\operatorname{Pic} X \oplus \operatorname{Pic} Y\right) \otimes \mathbb{Z}/n} \to \frac{\mathrm{H}^2(X \times Y, \mathbb{Z}/n)}{\mathrm{H}^2(X, \mathbb{Z}/n) \oplus \mathrm{H}^2(Y, \mathbb{Z}/n)} \to \frac{{}_n\mathrm{B}'(X \times Y)}{{}_n\mathrm{B}'(X) \oplus {}_n\mathrm{B}'(Y)} \to 0
$$

Combining (10) and (8) yields (7). □

Corollary 7. *Let X and Y be smooth curves over the algebraically closed field k of characteristic p. If n is relatively prime to p, then there are exact sequences*

$$
(11) \qquad 0 \to \operatorname{Pic} X \oplus \operatorname{Pic} Y \to \operatorname{Pic} X \times Y \to C \to 0
$$

$$
(12) \qquad 0 \to C/nC \to \mathrm{H}^1(X, \mathbb{Z}/n) \otimes \mathrm{H}^1(Y, \mathbb{Z}/n) \xrightarrow{\mu} {}_n\mathrm{B}(X \times Y) \to 0
$$

where C is defined by the first sequence.

Proof. For the smooth surface $X \times Y$, $\mathrm{B}(X \times Y) = \mathrm{B}'(X \times Y)$. For smooth curves the groups $\mathrm{H}^i(X, \mathbb{Z}/n)$ are free $\mathbb{Z}/n$-modules, $i > 0$. Over k the Brauer group of a curve is trivial. □

Corollary 8. *Let X and Y be projective nonsingular varieties over the algebraically closed field k of characteristic p. Assume n is relatively prime to p and either*

a. X and Y are both curves, or

b. n is a prime.

If $C = \mathrm{Pic}\,(X \times Y)/(\mathrm{Pic}\,X \oplus \mathrm{Pic}\,Y)$ *and*

$$\mu : K(X)^*/K(X)^{*n} \otimes K(Y)^*/K(Y)^{*n} \to \mathrm{B}\big(K(X \times Y)\big) \tag{13}$$

then $C/nC \cong \ker \mu$.

Proof. There is a natural injection

$$\mathrm{H}^1(X, \mathbb{Z}/n) \otimes \mathrm{H}^1(Y, \mathbb{Z}/n) \to K(X)^*/K(X)^{*n} \otimes K(Y)^*/K(Y)^{*n} \,.$$

Choose arbitrary open subsets U and V of X and Y respectively. Let

$$D = \mathrm{Pic}\,(U \times V)/(\mathrm{Pic}\,U \oplus \mathrm{Pic}\,V)\,.$$

The diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathrm{Pic}\,X \oplus \mathrm{Pic}\,Y & \longrightarrow & \mathrm{Pic}\,X \times Y & \longrightarrow & C & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle \alpha} & & \downarrow{\scriptstyle \beta} & & \downarrow{\scriptstyle \phi} & & \\ 0 & \longrightarrow & \mathrm{Pic}\,U \oplus \mathrm{Pic}\,V & \longrightarrow & \mathrm{Pic}\,U \times V & \longrightarrow & D & \longrightarrow & 0 \end{array} \tag{14}$$

commutes. Since α and β are surjective, ϕ is surjective. The diagram

$$
\begin{array}{ccccccc}
0 & \longrightarrow & C/nC & \longrightarrow & \mathrm{H}^1(X,\mathbb{Z}/n)\otimes \mathrm{H}^1(Y,\mathbb{Z}/n) & \longrightarrow & {}_n\mathrm{B}(X\times Y) \\
 & & \downarrow \sigma & & \downarrow \tau & & \downarrow \gamma \\
0 & \longrightarrow & D/nD & \longrightarrow & \mathrm{H}^1(U,\mathbb{Z}/n)\otimes \mathrm{H}^1(V,\mathbb{Z}/n) & \longrightarrow & {}_n\mathrm{B}(U\times V)
\end{array} \tag{15}
$$

commutes and τ and γ are one-to-one. Therefore σ is one-to-one, hence $C/nC \cong D/nD$. Taking the limit of D/nD over all U and V gives $\ker \mu$. □

Example 9. Let X be a projective nonsingular elliptic curve over the algebraically closed field k. Suppose $\operatorname{char} k = p$, $p \neq 2$. With notation taken from [16, IV.4], say $\tau = i$, $j = 1728$. Then $\operatorname{Pic} X = \mathbb{Z} \oplus (\mathbb{R}/\mathbb{Z})^2$. If C is as in (11), then $C = \operatorname{End}(X, P_0) \cong \mathbb{Z}[i]$. Thus $C/nC \cong (\mathbb{Z}/n)^2$. $\mathrm{H}^1(X,\mathbb{Z}/n) \cong {}_n\operatorname{Pic} X \cong (\mathbb{Z}/n)^2$. Applying Corollary 7 we get ${}_n\mathrm{B}(X\times X) \cong (\mathbb{Z}/n)^2$. Modulo p-groups, $\mathrm{B}(X \times X) \cong (\mathbb{Q}/\mathbb{Z})^2$.

Example 10. Let k be the complex number field. Let X be the complement of the curve $x^n = y^{n-1}z$ in the projective plane $\mathbb{P}^2$. Let Y be the affine nodal curve $y^2 = x^2(x+1)$. In [8] it was shown that $\mathrm{B}(X\times Y) \cong \mathbb{Z}/n$. One can compute $\mathrm{B}(X) = \mathrm{B}(Y) = 0$, $\operatorname{Pic} X = \mathbb{Z}/n$, $\mathrm{H}^1(X,\mathbb{Z}/n) = \mathbb{Z}/n$, $\mathrm{H}^2(X,\mathbb{Z}/n) = \mathbb{Z}/n$, $\mathrm{H}^1(Y,\mathbb{Z}/n) = \mathbb{Z}/n$, and in (6) $C = 0$. Applying Corollary 6 we see that the generator of $\mathrm{B}(X\times Y)$ is a cup product.

Corollary 11. *Let X and Y be connected schemes of finite type over the algebraically closed field k of characteristic p. Let $n > 1$ be relatively prime to p. Suppose $\mathrm{H}^i(X,\mathbb{Z}/n)$ is a free $\mathbb{Z}/n$-module for $i > 0$. If ${}_n\mathrm{B}(X) = {}_n\mathrm{B}'(X)$ and ${}_n\mathrm{B}(Y) = {}_n\mathrm{B}'(Y)$, then ${}_n\mathrm{B}(X\times Y) = {}_n\mathrm{B}'(X\times Y)$.*

Proof. From Corollary 6, ${}_n\mathrm{B}'(X\times Y)$ is generated by ${}_n\mathrm{B}(X)$, ${}_n\mathrm{B}(Y)$ and $\operatorname{im}\mu$. But these groups are subgroups of ${}_n\mathrm{B}(X\times Y)$. □

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DEPARTMENT OF MATHEMATICS, FLORIDA ATLANTIC UNIVERSITY, BOCA RATON, FLORIDA 33431